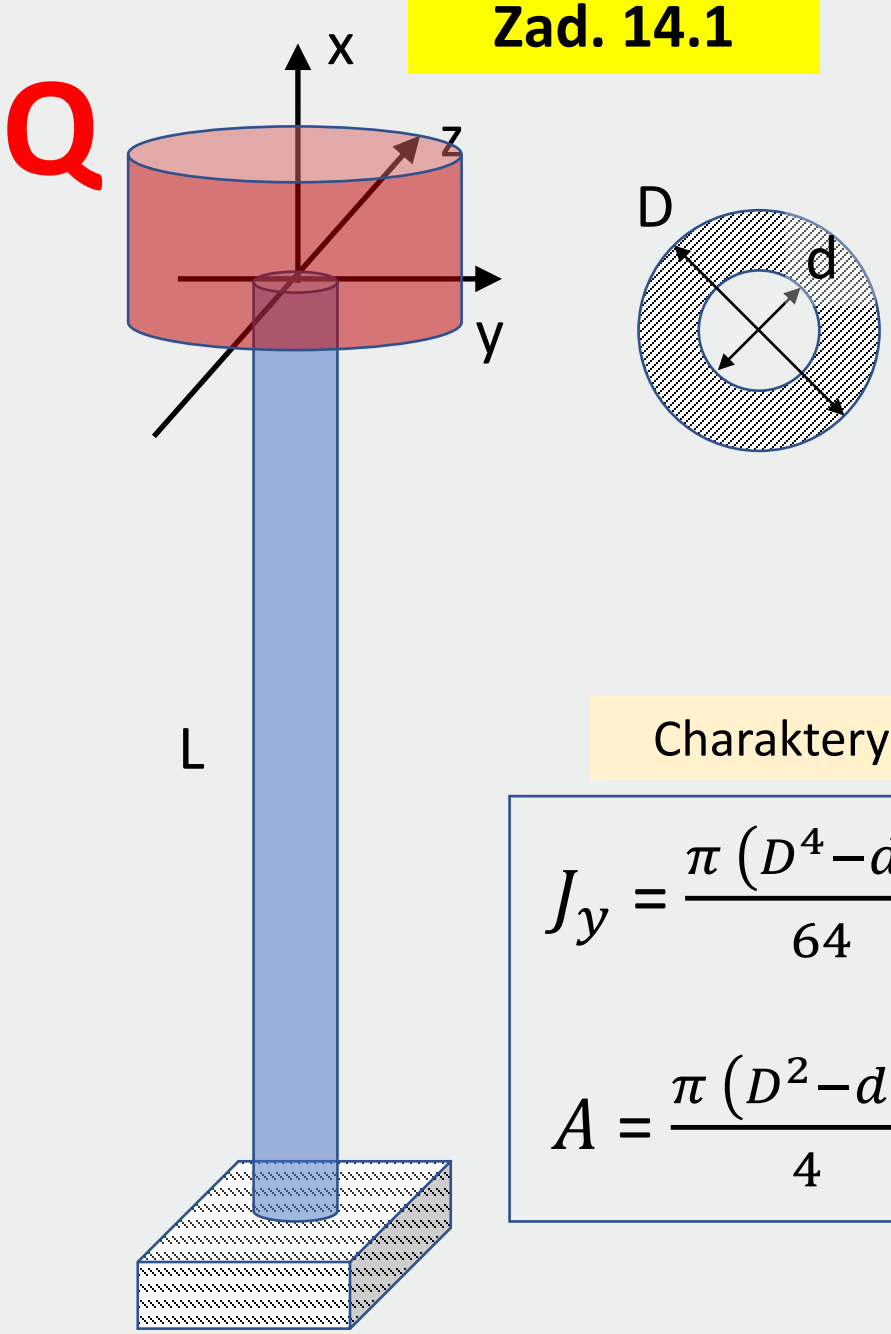
Wydział Mechaniczny Energetyki i Lotnictwa
Zakład Wytrzymałości Materiałów i Konstrukcji

Ćwiczenia 14

Wyboczenie pręta

Zad. 14.1



D = 10 cm , d = 4 cm , L = 3 m

E = $2 \cdot 10^5$ MPa , σ_{prop} = 220 MPa , n_{kr} = 5

Q_{max}=?

Smukłość graniczna:

$$\lambda_{gr} = \pi\sqrt{\frac{E}{\sigma_{prop}}} = \pi\sqrt{\frac{2 \cdot 10^5}{220}} = 95$$

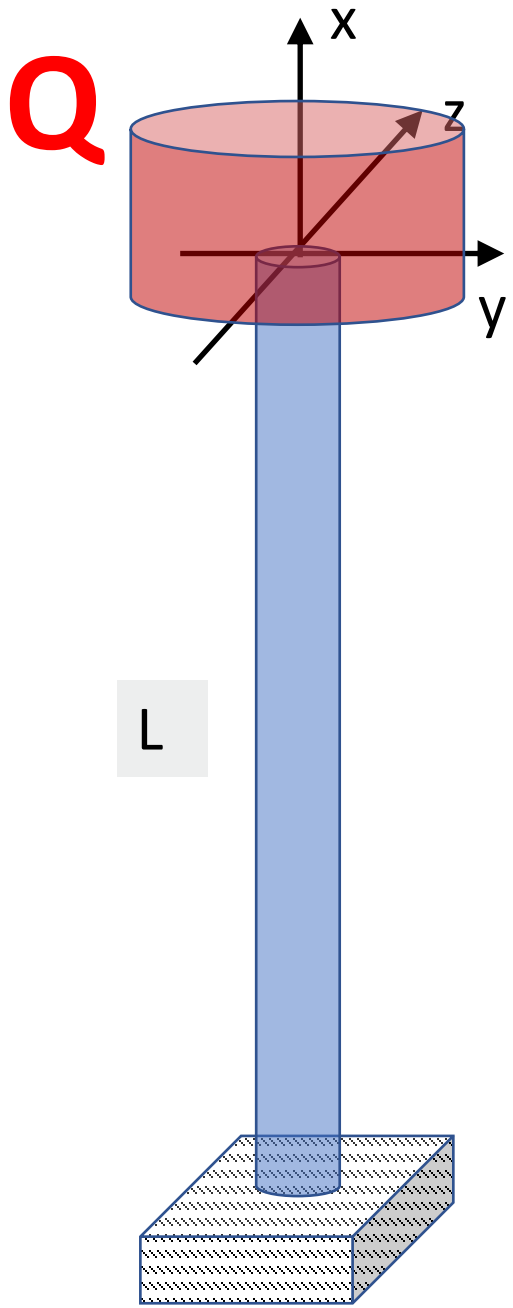
Charakterystyki przekroju:

$$J_y = \frac{\pi\left(D^4 - d^4\right)}{64} = 478\ cm^4$$

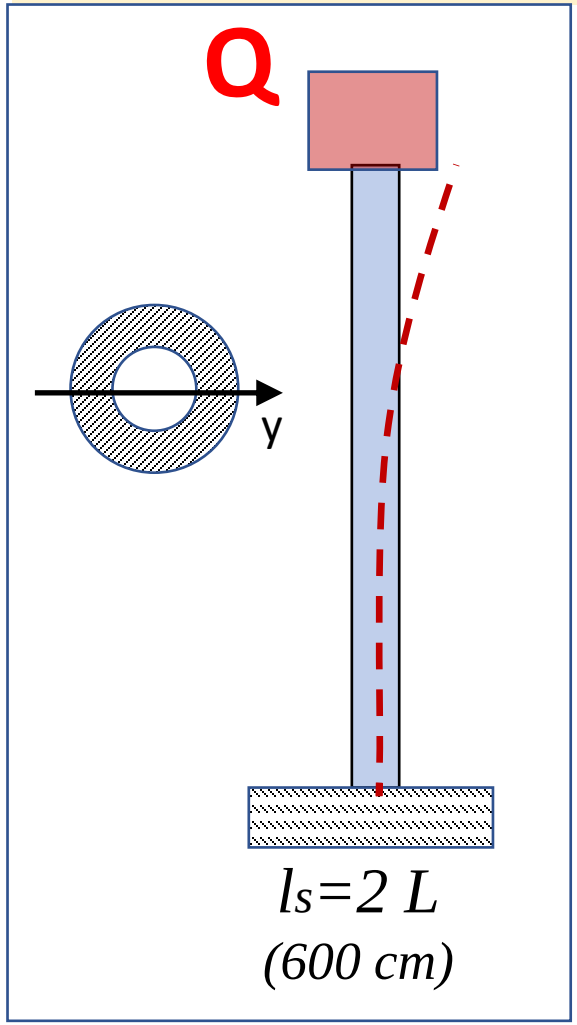
$$A = \frac{\pi\left(D^2 - d^2\right)}{4} = 66\ cm^2$$

Promień bezwładności:

$$i_y = \sqrt{\frac{J_y}{A}} = \sqrt{\frac{478}{66}} = 2.69\ cm$$



Określenie możliwej postaci wyboczenia:

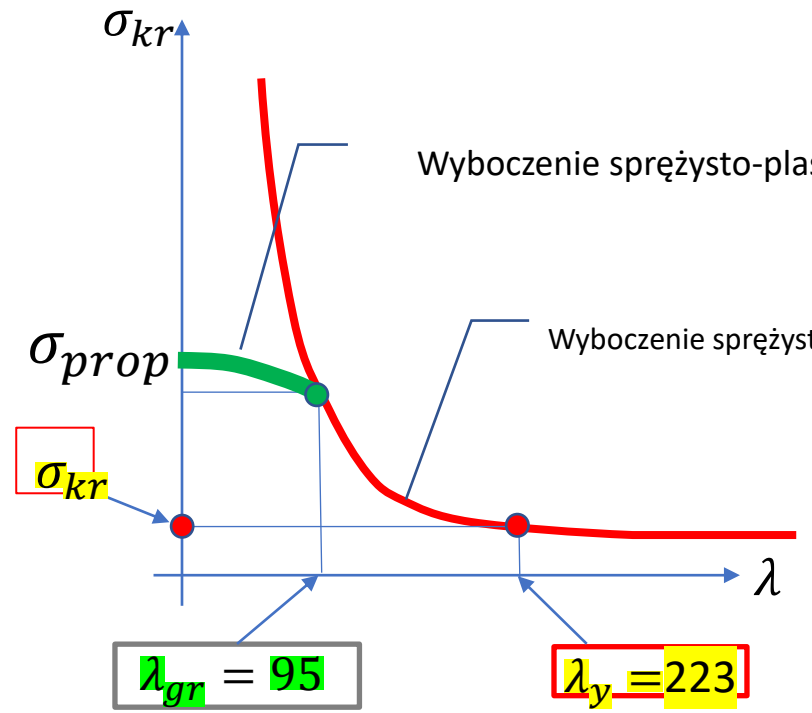


$\lambda_{gr} = 95$

$i_y = 2.69\ cm$

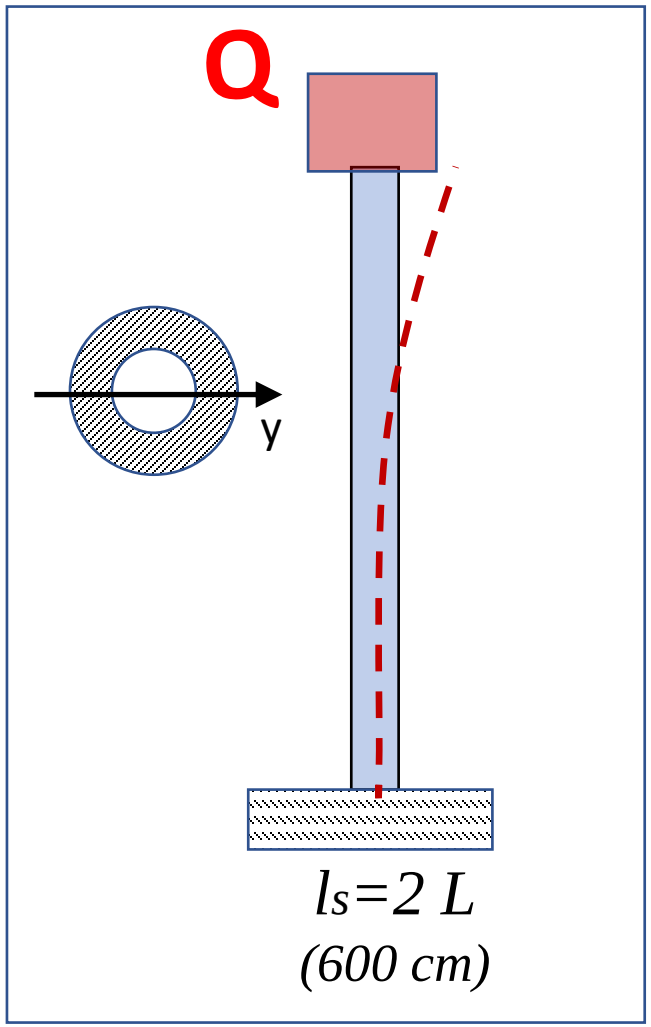
Określenie smukłości:

$$\lambda_y = \frac{l_s}{i_y} = \frac{600}{2.69} = 223$$



$\lambda_y > \lambda_{gr}$

Można stosować wzór Eulera



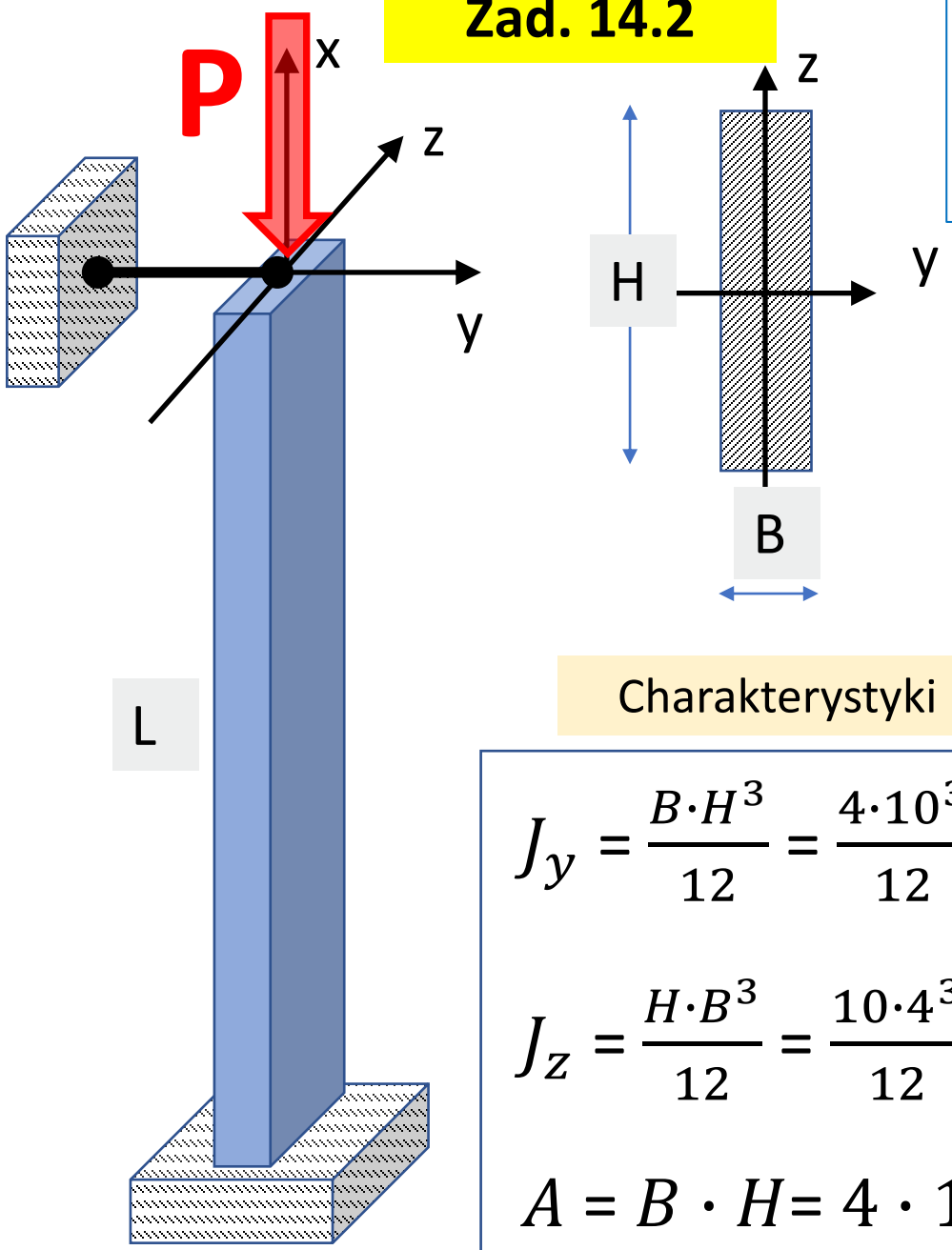
Określenie siły krytycznej i dopuszczalnej

$$P_{kr} = \frac{\pi^2 E J_y}{l_s^2} = \frac{\pi^2 \, 2 \cdot 10^{11} \cdot 478 \cdot 10^{-8}}{6^2} = 262093 \; N$$

$$\boldsymbol{P}_{dop} = \frac{\boldsymbol{P}_{kr}}{\boldsymbol{n}_{kr}} = \frac{262093}{5} = 52418 \; N$$

$$\boldsymbol{Q}_{max} \leq \boldsymbol{P}_{dop} = 52.4 \text{ kN}$$

Zad. 14.2



H = 10 cm , B = 4 cm , L = 3 m

$E = 7 \cdot 10^4$ MPa , $R_{0,2} = 280$ MPa , $n_{kr} = 3$

P_{max}=?

Smukłość graniczna:

$$\lambda_{gr} = \pi \sqrt{\frac{2E}{R_{0,2}}} = \pi \sqrt{\frac{2 \cdot 7 \cdot 10^4}{280}} = 70$$

Charakterystyki przekroju:

$$J_y = \frac{B \cdot H^3}{12} = \frac{4 \cdot 10^3}{12} = 333\ cm^4$$

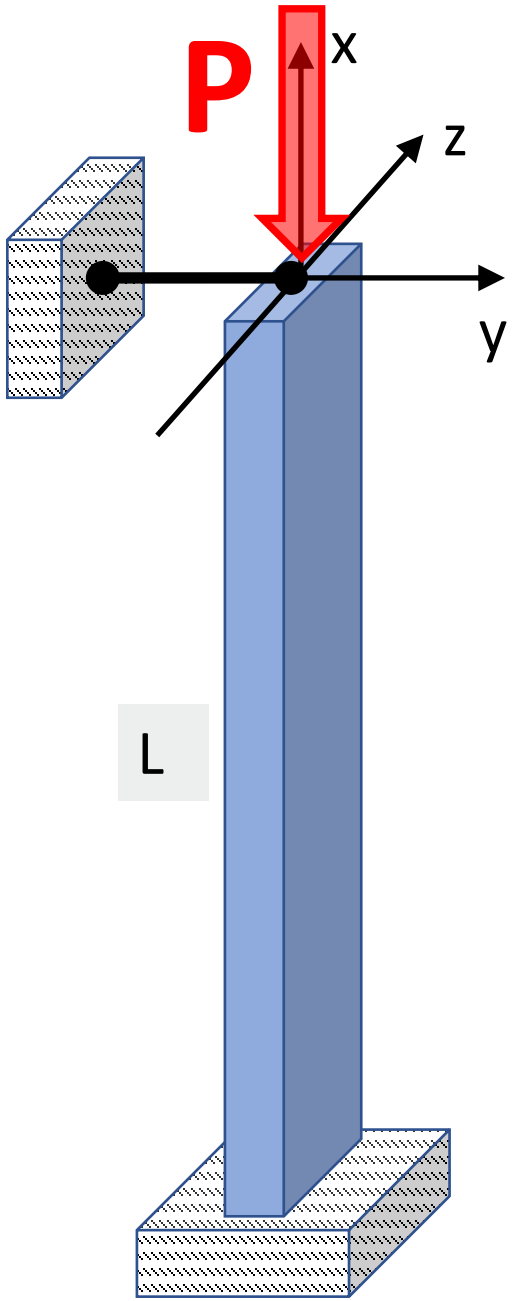
$$J_z = \frac{H \cdot B^3}{12} = \frac{10 \cdot 4^3}{12} = 53\ cm^4$$

$$A = B \cdot H = 4 \cdot 10 = 40\ cm^2$$

Promienie bezwładności:

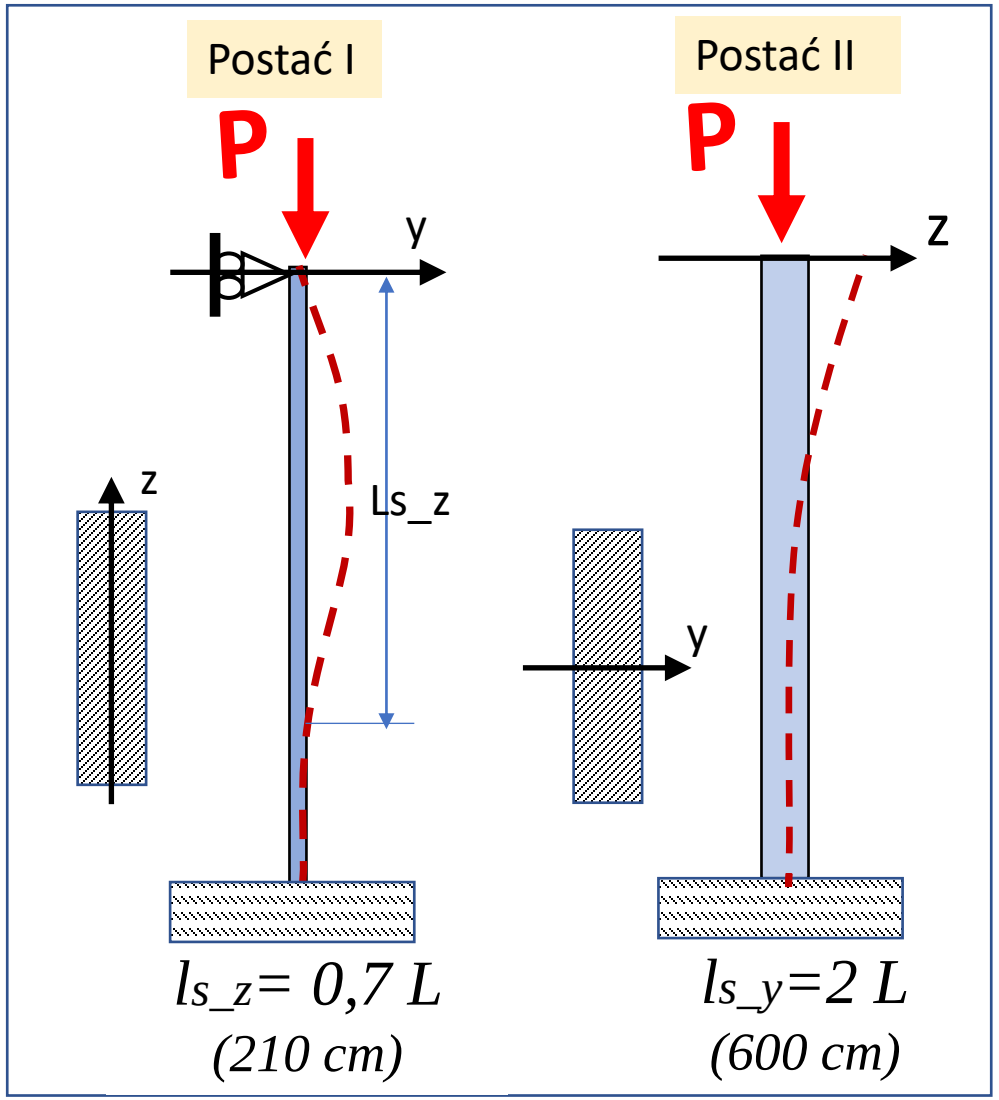
$$i_y = \sqrt{\frac{J_y}{A}} = \sqrt{\frac{333}{40}} = 2.89\ cm$$

$$i_z = \sqrt{\frac{J_z}{A}} = \sqrt{\frac{53}{40}} = 1.15\ cm$$



$\lambda_{gr} = 70$ $i_y = 2.89\ cm$ $i_z = 1.15\ cm$

Określenie możliwych postaci wyboczenia:



Określenie smukłości:

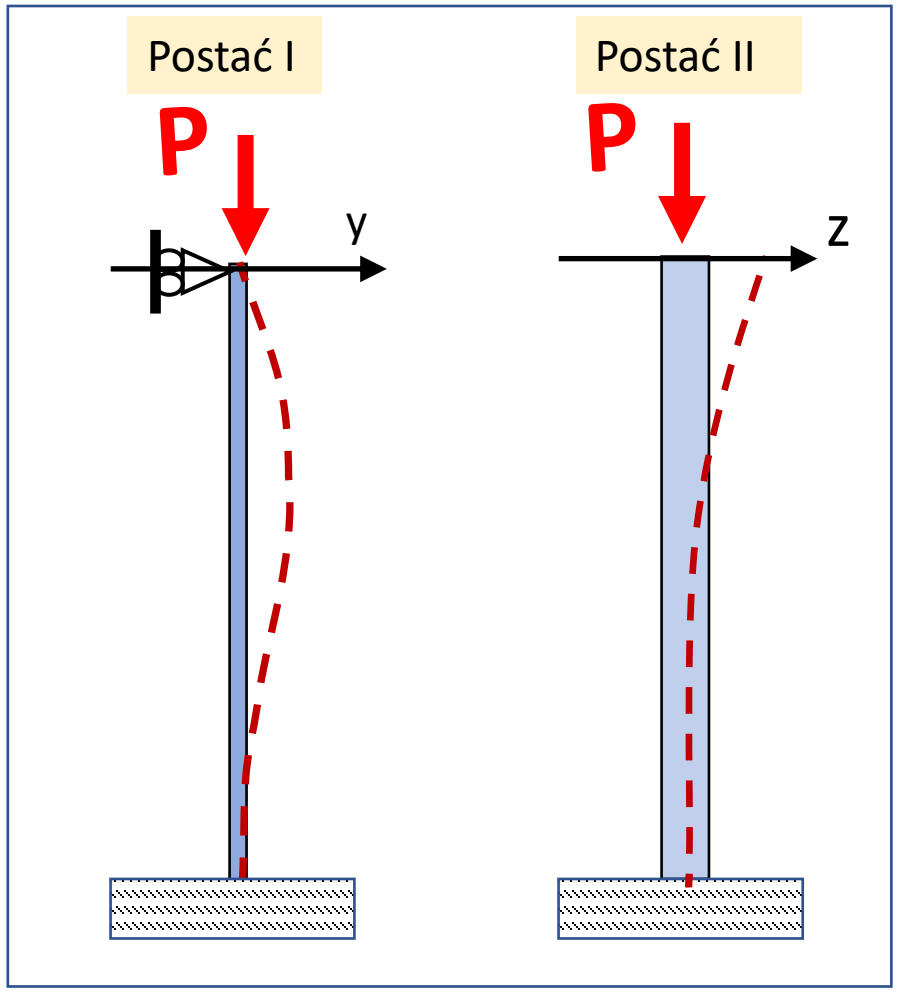
$$\lambda_y = \frac{l_{s\_y}}{i_y} = \frac{600}{2.89} = 208$$

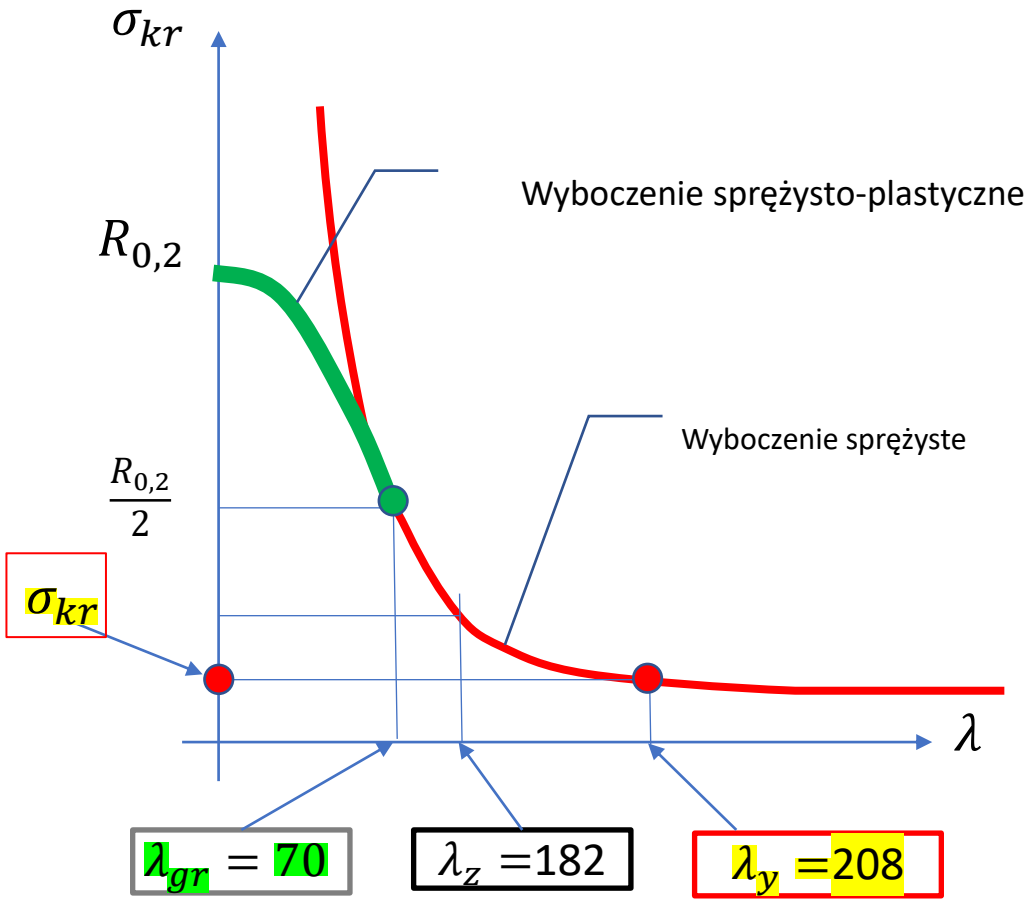
$$\lambda_z = \frac{l_{s\_z}}{i_z} = \frac{210}{1.15} = 182$$

$$\lambda_y > \lambda_z > \lambda_{gr}$$

Można stosować wzór Eulera

Określenie postaci utraty stateczności





$$\lambda_y > \lambda_z > \lambda_{gr}$$

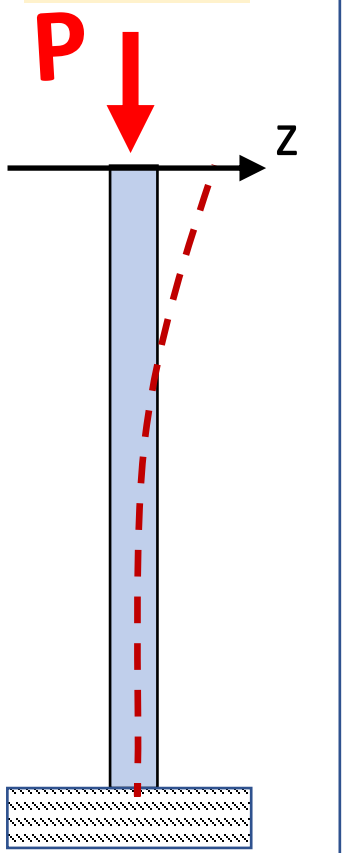
→ Pręt utraci stateczność w postaci II

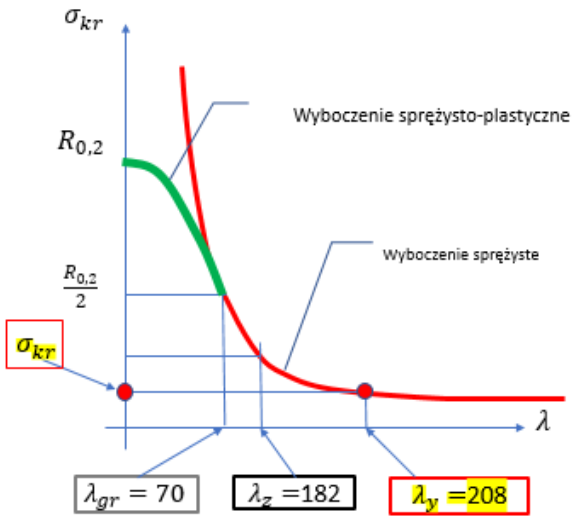
Można stosować wzór Eulera

Pręt utraci stateczność w postaci II

Określenie siły krytycznej i dopuszczalnej

Postać II





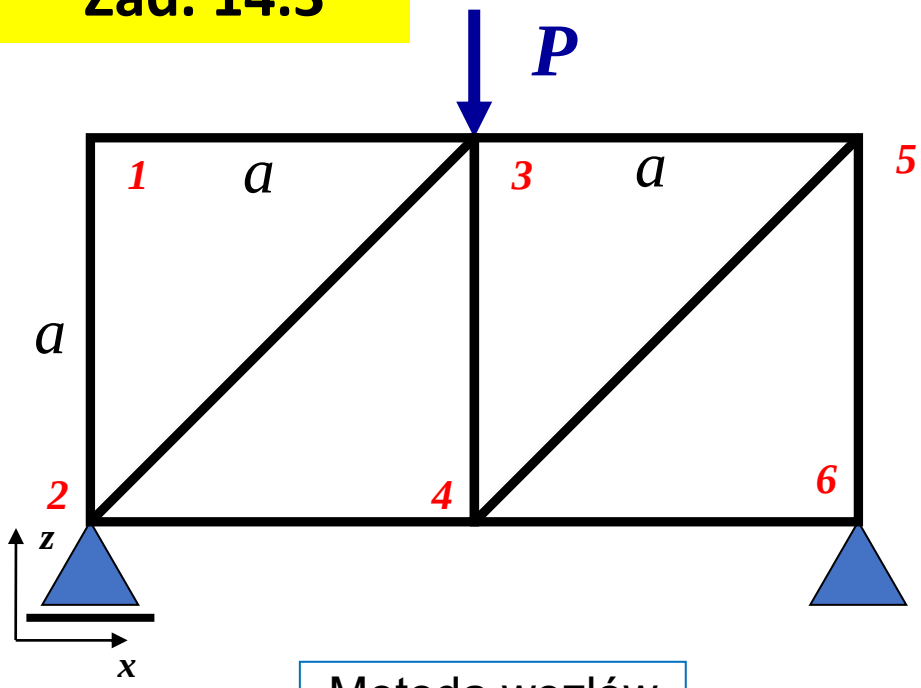
$$P_{kr} = \frac{\pi^2 E J_y}{l_{s\_y}^2} = \frac{\pi^2 \cdot 7 \cdot 10^{10} \cdot 333 \cdot 10^{-8}}{6^2} = 63900\ N$$

$$\boldsymbol{P_{dop}} = \frac{\boldsymbol{P_{kr}}}{\boldsymbol{n_{kr}}} = \frac{63900}{3} = 21301\ N$$

$$\boldsymbol{P_{max}} \leq \boldsymbol{P_{dop}} = 21.3\ \text{kN}$$

Zad. 14.3

Kratownica płaska



Dane:

$P = 2000\ N$

$a = 1\ m$

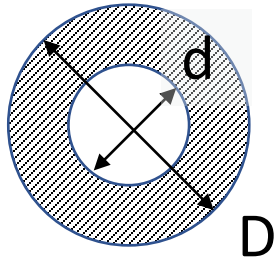
$E = 2 \cdot 10^5\ MPa$

$\lambda_{gr} = 100$

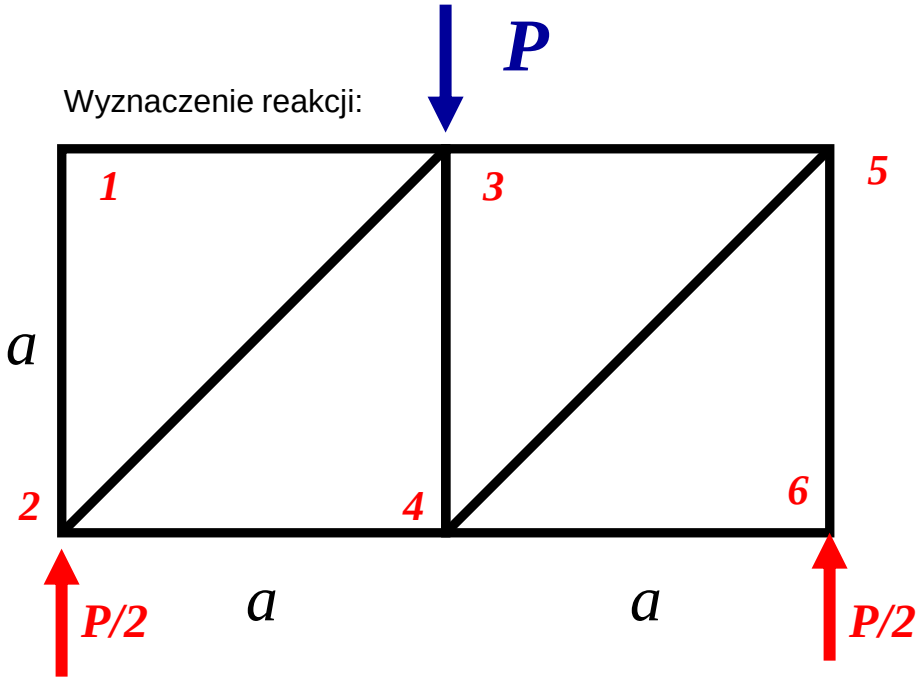
$n_{kr} = 5$

$D = 2d$

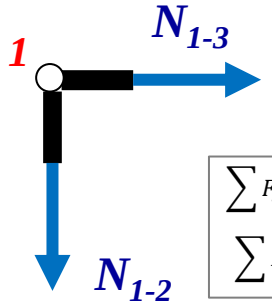
$d = ?$



Wyznaczenie reakcji:



Metoda węzłów

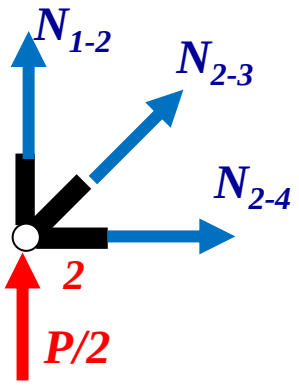


$\sum F_x = 0$

$\sum F_z = 0$

$N_{1-3}=0$

$N_{1-2}=0$

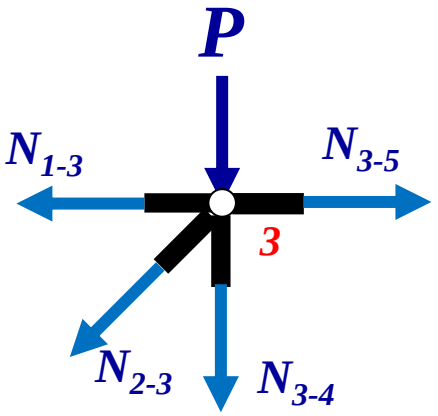


$N_{2-4} + N_{2-3} \cos 45°=0$

$½P + N_{2-3} \sin 45°=0$

$N_{2-4}= ½P$

$N_{2-3}=-½\sqrt{2}P$



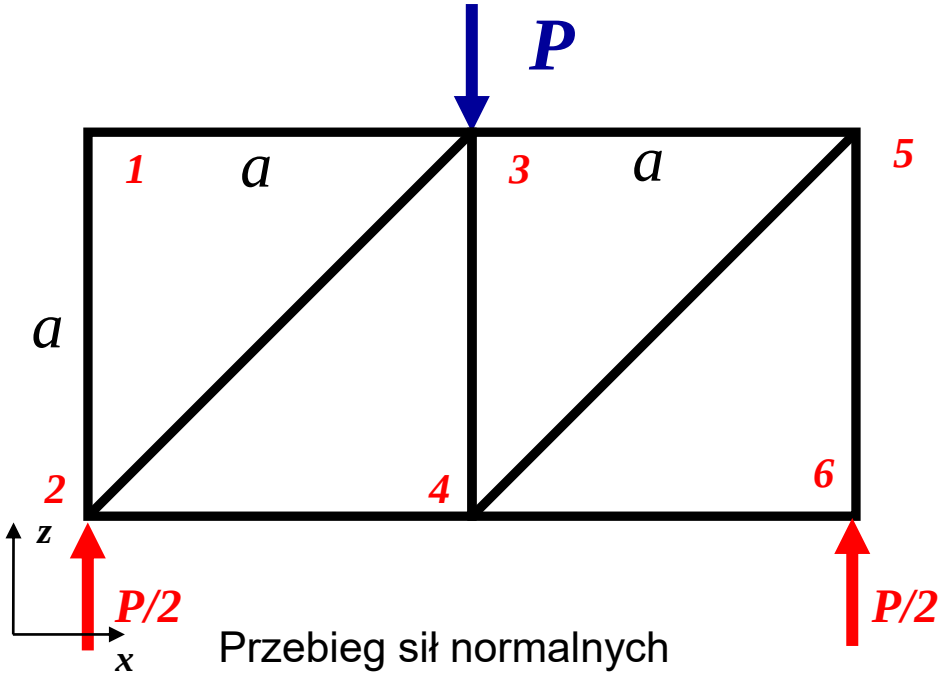
$N_{3-5} - N_{2-3} \cos 45°=0$

$-P - N_{2-3} \sin 45° - N_{3-4} =0$

$N_{3-5}= -½P$

$N_{3-4}= -½P$

i.t.d



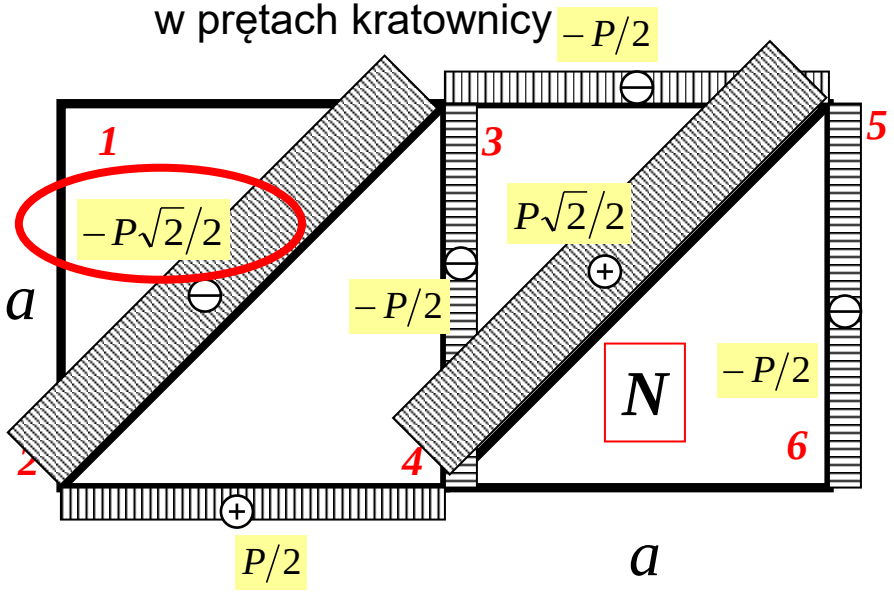
N_{1-3}=0
N_{1-2}=0

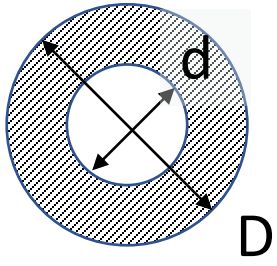
N_{2-4}= ½P
N_{2-3}=−½√2P

N_{3-5}= −½P
N_{3-4}= −½P

i.t.d

Przebieg sił normalnych w prętach kratownicy





Charakterystyki przekroju:

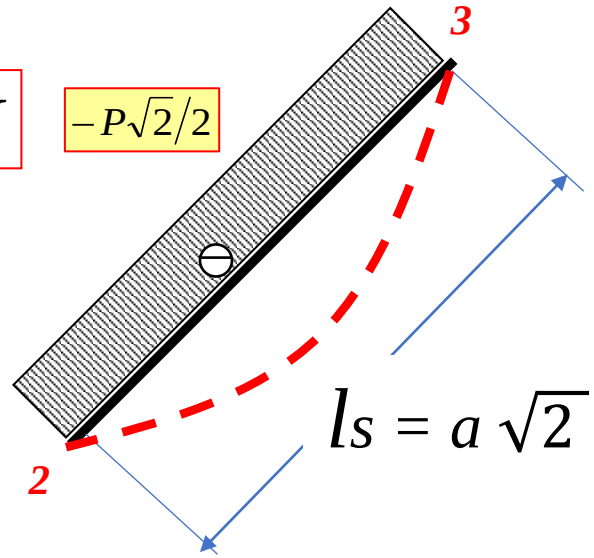
$$J_y = \frac{\pi\left(D^4-d^4\right)}{64} = \frac{15\,\pi d^4}{64}$$

$$A = \frac{\pi\left(D^2-d^2\right)}{4} = \frac{3\,\pi d^2}{4}$$

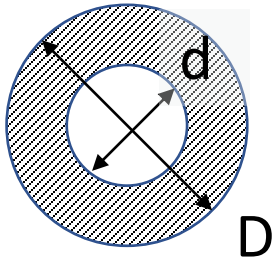
Promień bezwładności:

$$i_y = \sqrt{\frac{J_y}{A}} = \sqrt{\frac{15}{48}}\,d$$

Zagrożony ściskaniem pręt



Postać wyboczenia



$$J_y = \frac{15\,\pi d^4}{64}$$

$$i_y = \sqrt{\frac{15}{48}}\,d$$

$$P_{kr} = \frac{\pi^2 E\, J_y}{l_s^2} = \frac{\pi^2 E \cdot \frac{15\,\pi d^4}{64}}{(a\sqrt{2})^2}$$

$$P_{dop} = \frac{P_{kr}}{n_{kr}} = \frac{\pi^2 E \cdot \frac{15\,\pi d^4}{64}}{n_{kr}\, 2\, a^2} \geq P\,\frac{\sqrt{2}}{2}$$

$$d^4 \geq \frac{32\sqrt{2}\, n_{kr} P\, 2\, a^2}{15\,\pi^3 E} = \frac{32\sqrt{2}\cdot 5\cdot 2000\cdot 2\cdot 1^2}{15\,\pi^3 2\cdot 10^{11}}$$

$$d \geq 0.00993\ m = 9.93\ mm \rightarrow$$

$$i_y = \sqrt{\frac{15}{48}}\,d = 5.55\ \text{mm}$$

Sprawdzenie smukłości:

$$\lambda_y = \frac{l_s}{i_y} = \frac{1000\sqrt{2}}{5.55} = 254 \geq \lambda_{gr}$$

OK